A MultiPhase Power Flow Model for μGrid Analysis

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Abstract

This paper presents a new advanced model of an electric power system with distributed energy sources forming a microgrid (μGrid). The μGrid is a radial or networked low voltage distribution system with distributed sources. Each source is interfaced to the system via converters. The DC bus of the converter may have energy storage capability via large capacitors or batteries. The μGrid load consists of both single and three phase loads resulting in unbalanced operating conditions. The μGrid circuits may be three-wire, four-wire or five-wire. The grounding of the system may be single point or multi-point. The analysis of this system requires a new approach. This paper presents a new method for modeling and analysis of this system. The approach consists of two steps: (a) modeling each component of the system via a set of quadratic equations no matter how complex the nonlinearities of the model are and (b) a Newton's method for the solution of the overall network equations. The method is extremely efficient and robust. The proposed method can accommodate various control modes of micro-sources. Examples of these controls are given in the paper.

1. Introduction

The concept of the μGrid was introduced by the DoE as a distribution system with distributed energy sources (microturbines, fuel cells, photovoltaics, etc.). The μGrid may be connected to a 35kV or 25kV or 13.8kV distribution system. It includes the step-down transformers and the 480V/208V system. The 480V or 208V system (secondary distribution system) may be radial or networked. Therefore the topology of the μGrid comprises the step down transformers, the 480V or 208V circuits – radial or networked, the electric loads and micro-sources (Distributed Energy Sources) that may be distributed along the 480V or the 208V system. Pictorial views of the single line diagram of a radial and networked system are illustrated in Figures 1 and 2 respectively.

The design of this system may include three-wire, four-wire and or five-wire circuits in accordance to the National Electrical Code (NEC). The operation of this system in general will be characterized with unbalance and by the fact that most of the distributed resources are inertia-less (because they are interfaced to the system via inverters and typically without available storage). In addition, since the interface to the grid is via converters a multiplicity of control functions can be anticipated, such as control of imbalances, control of power factor, etc. At the present time we know very little about the performance of such a system and what will be the effect of complex interactions among various controllers. Research projects are underway to quantify the performance of such systems and to utilize the benefits of the μGrid.

In this paper we address the issue of steady state operation of the μGrid. Specifically, we propose a new multiphase power flow analysis method that provides exact solution to the operation of the μGrid under steady state conditions and satisfying all the prevailing system constraints and imposed controlled laws.

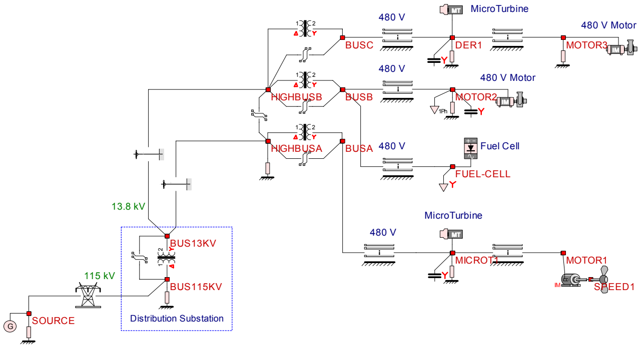


Figure 1 Single Line Diagram of a Radial μGrid - Conceptual

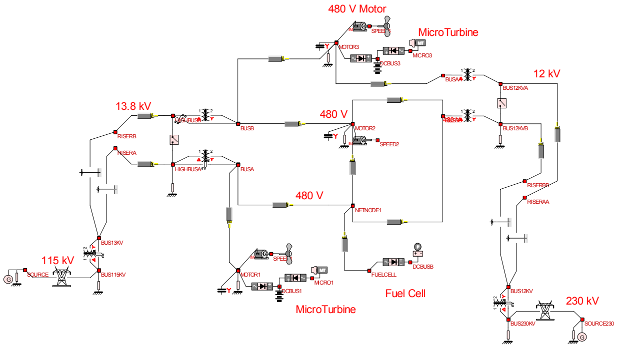


Figure 2 Single Line Diagram of a Networked µGrid - Conceptual

The paper describes the models of various microsources and the various control options. For example, a microsource may control the negative sequence voltages at the point of common coupling, it may control the power factor, the voltage magnitude, etc. These controls are explicitly incorporated in the model. Subsequently, connectivity constraints are applied to the device models, yielding the system network equations. These equations are solved via Newton's method. The paper provides convergence data for specific microgrid systems. The paper also provides examples of microsources that balance the operation of the microgrid by appropriate selection of their controls. The proposed method has been implemented. The resulting computer program has been named mGrid.

2. Description of the Proposed Model

The proposed multiphase power flow method follows an object-oriented approach. Each µGrid component is represented by a component object, which contains a *full 3-phase* representation of the device equivalent circuit, as well as control laws. Consider for example a micro source model, whose equivalent circuit is illustrated in Figure 3.

The circuit of Figure 3 is represented by a set of equations derived by circuit analysis (Kirchoff's laws) plus a number of control equations, specifically:

$$\tilde{I}_a = jb(\tilde{V}_a - \tilde{V}_n) - Ie^{j\phi} \tag{1}$$

$$\tilde{I}_b = jb(\tilde{V}_b - \tilde{V}_n) - Ie^{j(\phi - \frac{2\pi}{3})}(1 + jx_1)$$

$$\tilde{I}_c = jb(\tilde{V}_c - \tilde{V}_n) - Ie^{j(\phi - \frac{4\pi}{3})}(1 + jx_2)$$

$$\tilde{I}_n = -jb(\tilde{V}_a + \tilde{V}_b + \tilde{V}_c - 3\tilde{V}_n)$$

$$\mathrm{Re}\left[(\tilde{V}_a - \tilde{V}_n)\tilde{I}_a^* + (\tilde{V}_b - \tilde{V}_n)\tilde{I}_b^* + (\tilde{V}_c - \tilde{V}_n)\tilde{I}_c^*\right] = -P$$

$$\mathrm{Im}\left[(\tilde{V}_a - \tilde{V}_n)\tilde{I}_a^*\right] = 0.0$$

$$\mathrm{Im}\left[(\tilde{V}_a - \tilde{V}_n)\tilde{I}_a^*\right] = 0.0$$

$$\mathrm{Im}\left[(\tilde{V}_a - \tilde{V}_n)\tilde{I}_a^*\right] = 0.0$$

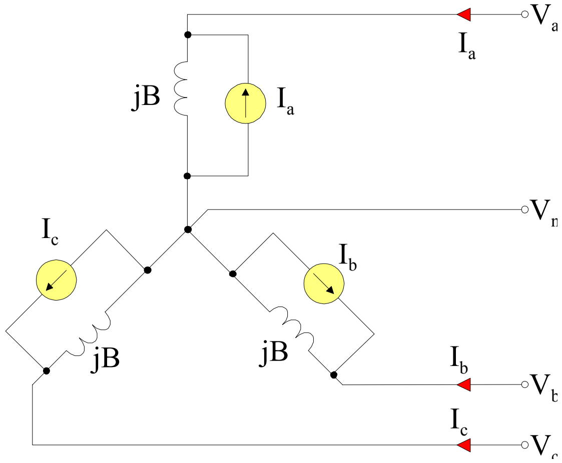


Figure 3: Micro-Source Equivalent Circuit

Note that in this specific example, the control equations set the total real power to a specified value (P), and the reactive power is set to zero, individually on each phase. The modeling methodology allows for including various such control schemes with each model object, thus providing flexibility in capturing the actual physical device behavior. The state variables for this model are the voltage phasors, $\tilde{V}_a$, $\tilde{V}_b$, $\tilde{V}_c$, $\tilde{V}_n$, the internal source current magnitude I, and phase angle ϕ, and the real variables x_1 and x_2. Note that the state variables are defined so that the number of state variables equals the number of equations, resulting into a consistent equation system. In this morel, the state variables x_1 and x_2 simulate the device capability to modify the phase angles of the B and C phase internal current sources in order to provide individual phase reactive power control.

The equations of each model are subsequently quadratized resulting in the following form:

$$\begin{bmatrix}\tilde{I}^k\\ 0\end{bmatrix} = y^k_{eq\_cmpx}\begin{bmatrix}\tilde{V}^k\\ \tilde{Y}^k\end{bmatrix} + F\left\{\begin{bmatrix} x^{k^T} f^k_{eq\_real1} x^k \\ x^{k^T} f^k_{eq\_real2} x \\ \vdots \end{bmatrix}\right\} - b^k_{eq\_cmpx} \qquad (2)$$

Where $\tilde{I}^k$: vector of terminal currents,
$\tilde{V}^k$: vector of terminal voltages,
$\tilde{Y}^k$: vector of device internal state variables,
$\tilde{X}^k = \left[\tilde{V}^k \quad \tilde{Y}^k\right]^T$,
$x^k = vector\ \tilde{X}^k\ in\ cartesian\ Coordinates$
and $y^k_{eq\_cmpx}$, $b^k_{eq\_cmpx}$, and $f^k_{eq\_real}$ are matrices with appropriate dimensions. $F(\bullet)$ denotes a function mapping from a real vector to a complex vector. Note that this form includes two sets of equations, which are named *external equations* and *internal equations* respectively. The terminal currents appear only in the external equations. Similarly, the device states consist of two sets: *external states* (i.e. terminal voltages, $\tilde{V}^k$) and *internal states* $\tilde{Y}^k$. The set of equations (2) is consistent in the sense that the number of external states and the number of internal states equals the number of external and internal equations respectively. These equations resemble the Norton equivalent circuit equations for electrical components. For this reason we have named this form the Generalized Norton Form (GNF).

The entire network equations are obtained by application of the connectivity constraints among component objects. For electrical circuits, the connectivity constraints are obtained by applying Kirchoff's current law at each node of the system. This procedure yields a set of equations, which are combined with the component object internal equations resulting in the set of equations of the form:

$$\sum_k A^k \tilde{I}^k = 0 \qquad (3)$$

[plus internal equations of all devices]

where $\tilde{I}^k$ is component *k* terminal currents composed of the currents at the composite nodes *k1*, *k2*, etc. A^k is a component incidence matrix with:

$\left\{A^k_{ij}\right\}$ =1, if terminal *j* of component *k* is connected to node *i* = 0, otherwise

Let $\tilde{V}$ be the vector of voltages at all the nodes of the system grouped by composite nodes. Then, the following relationship holds:

$$\tilde{V}^k = (A^k)^T \tilde{V} \qquad (4)$$

where $\tilde{V}^k$ is component k terminal voltages. Upon substitution of device equations (2) and incidence equations (4), equation (3) becomes a set of quadratic equations:

$$\tilde{Y}\tilde{X} + F\begin{bmatrix} x^T f_1 x \\ x^T f_2 x \\ \vdots \end{bmatrix} - \tilde{B} = 0 \qquad (5)$$

where $\tilde{X}$ is the vector of all the state variables and $\tilde{Y}, f, B$ are matrices with appropriate dimensions. The simultaneous solution of these equations is obtained via Newton's method described next.

The numerical algorithm for solving the network equations (5) consists of two steps. First, we convert the network equations (5) into Cartesian coordinates by simply replacing each complex variable with its Cartesian form and separating the real and imaginary parts of the complex equations. The procedure is equivalent with replacing each element in $\tilde{Y}$ with its corresponding 2×2 Hermetian matrix. In particular, $\tilde{Y}_{ij}$ is replaced by:

$$\begin{bmatrix} \tilde{Y}^r_{ij} & -\tilde{Y}^i_{ij} \\ \tilde{Y}^i_{ij} & \tilde{Y}^r_{ij} \end{bmatrix} \qquad (6)$$

where superscript *r* denotes real part and superscript *i* denotes imaginary part. Then, equation (5) is transformed into Equation (7) below:

$$Y_{real}\, x + \begin{bmatrix} x^T f_1 x \\ x^T f_2 x \\ \vdots \end{bmatrix} - B_{real} = 0 \qquad (7)$$

Equation (7) is solved using Newton's method. Specifically, the solution is given by the following algorithm:

$$x^{\nu+1} = x^{\nu} - J^{-1}\left\{ Y_{real}\, x^{\nu} + \begin{bmatrix} x^{\nu^T} f_1 x^{\nu} \\ x^{\nu^T} f_2 x^{\nu} \\ \vdots \end{bmatrix} - B_{real} \right\} \qquad (8)$$

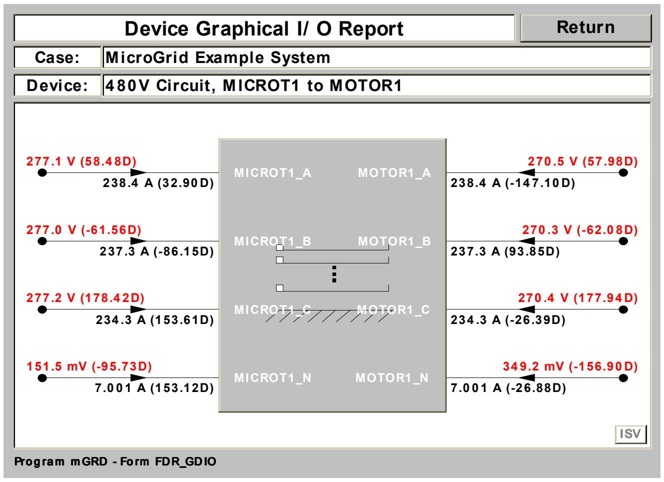
where ν is the iteration step number; J is the Jacobian matrix of equation (7). In particular, the Jacobian matrix takes the following form:

$$J = Y_{real} + \begin{bmatrix} x^{\nu^T}(f_1 + f_1^T) \\ x^{\nu^T}(f_2 + f_2^T) \\ \vdots \end{bmatrix} \qquad (9)$$

It is important to note that Newton's method applied to a set of quadratic equations guarantees quadratic convergence. In fact, this algorithm (8) converges in only two or three iterations.

3. Example Test System

The proposed method has been applied to an example µGrid system, which is illustrated in Figure 1. The system comprises a number of three-wire, four-wire and five-wire circuits as well as several micro-sources (namely, two micro-turbines and a fuel cell), and three induction motors. The system is shown connected to the power grid via a 13.8 kV substation, but can also operate isolated from the power grid. After specifying the micro-source operating controls, and a solution is computed, any desired electrical quantity can be readily computed, such as node voltages, currents, and power flows. Note that the methodology follows a physical modeling approach (with full 3-phase representation), and thus voltages, currents and power flows can be reported on any individual phase as well as neutral and ground wires. For example, Figures 4 and 5 report the voltages and currents at the terminals of a 3-phase distribution line and induction motor 1. Note the capability of the method to compute the voltages and currents in the neutral as well as in the ground conductor. The system exhibits slight imbalances on the phase voltages and currents.

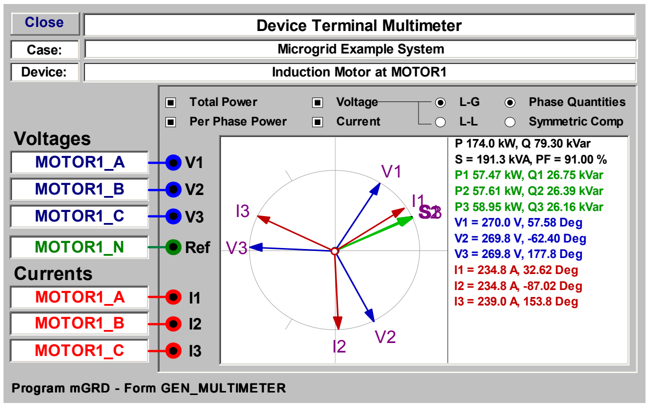


Case: MicroGrid Example System

Device: 480V Circuit, MICROT1 to MOTOR1

Program mGRD - Form FDR_GDIO

Figure 4: Distribution Line MICROT1-MOTOR1 Voltages and Currents



Case: Microgrid Example System

Device: Induction Motor at MOTOR1

☒ Total Power
☒ Per Phase Power
☒ Voltage
☒ Current
● L-G
○ L-L
● Phase Quantities
○ Symmetric Comp

P 174.0 kW, Q 79.30 kVar
S = 191.3 kVA, PF = 91.00 %
P1 57.47 kW, Q1 26.75 kVar
P2 57.61 kW, Q2 26.39 kVar
P3 58.95 kW, Q3 26.16 kVar
V1 = 270.0 V, 57.58 Deg
V2 = 269.8 V, -62.40 Deg
V3 = 269.8 V, 177.8 Deg
I1 = 234.8 A, 32.62 Deg
I2 = 234.8 A, -87.02 Deg
I3 = 239.0 A, 153.8 Deg

Program mGRD - Form GEN_MULTIMETER

Figure 5: Induction Motor 1 Voltages and Currents

4. Applications

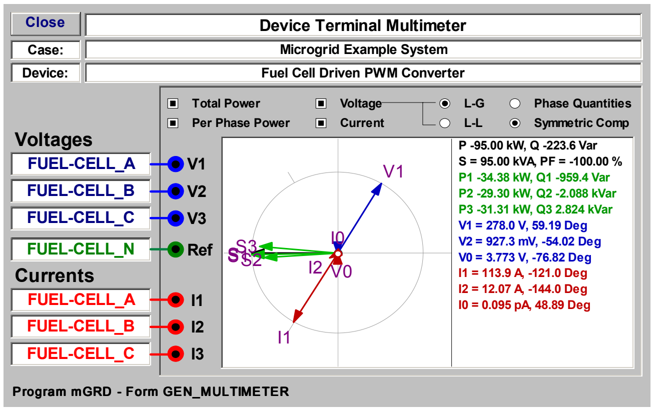
The proposed methodology can be utilized in several microgrid analysis applications. In the next sections we present examples of such applications, namely an analysis of the effects of various micro-source controllers, safety analysis, asymmetry and unbalance analysis, and objectionable current analysis.

Effects of Converter Controls: Presently available micro-sources are interfaced to the microgrid via inverters, which provide several control options, such as:

- Unity power factor,
- Reduction of voltage asymmetry,
- Reduction of negative sequence voltages, etc.

It is of interest to evaluate the impact of such specific control functions on the performance of the individual component, as well as the entire µGrid system.

Two example cases are presented next, both based on the mGrid system of Figure 1. I the first example case the controls of the PWM inverter (fed by a fuel cell) were set in PV mode, with P set to 95 kW and V set to 278 Volts. Figure 6 shows the PWM inverter device terminal voltages and currents obtained after the solution of this system. Note that the power output and terminal voltage closely match the control settings.

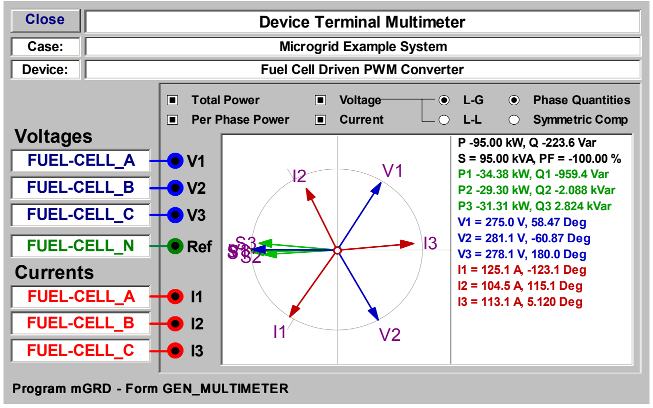


Case: Microgrid Example System
Device: Fuel Cell Driven PWM Converter

☑ Total Power ☑ Voltage ● L-G ○ Phase Quantities
☑ Per Phase Power ☑ Current ○ L-L ● Symmetric Comp

P -95.00 kW, Q -223.6 Var
S = 95.00 kVA, PF = -100.00 %
P1 -34.38 kW, Q1 -959.4 Var
P2 -29.30 kW, Q2 -2.088 kVar
P3 -31.31 kW, Q3 2.824 kVar
V1 = 278.0 V, 59.19 Deg
V2 = 927.3 mV, -54.02 Deg
V0 = 3.773 V, -76.82 Deg
I1 = 113.9 A, -121.0 Deg
I2 = 12.07 A, -144.0 Deg
I0 = 0.095 pA, 48.89 Deg

(a)



Case: Microgrid Example System
Device: Fuel Cell Driven PWM Converter

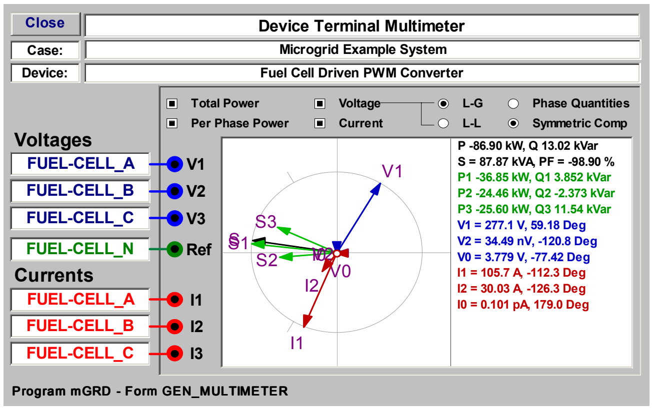
☑ Total Power ☑ Voltage ● L-G ● Phase Quantities
☑ Per Phase Power ☑ Current ○ L-L ○ Symmetric Comp

P -95.00 kW, Q -223.6 Var
S = 95.00 kVA, PF = -100.00 %
P1 -34.38 kW, Q1 -959.4 Var
P2 -29.30 kW, Q2 -2.088 kVar
P3 -31.31 kW, Q3 2.824 kVar
V1 = 275.0 V, 58.47 Deg
V2 = 281.1 V, -60.87 Deg
V3 = 278.1 V, 180.0 Deg
I1 = 125.1 A, -123.1 Deg
I2 = 104.5 A, 115.1 Deg
I3 = 113.1 A, 5.120 Deg

(b)

Figure 6: Voltage, current and power flow at the fuel cell PWM converter output terminals with PV control mode. (a) Symmetric Components, (b) Phase Quantities.

In the next example case, the controls of the PWM inverter were set for negative sequence control. The P and V settings were the same as for case 1. The PWM inverter terminal conditions for this case are illustrated in Figure 7. Note that the negative sequence voltage has dropped to practically zero (34.5 nano-volts). Note also that in order for the controller to control the negative sequence, the imbalance among the phases has increased. Notice that there is a 48 A different between phases A and B (while in the previous control case there was only 20 A difference between phases A and B). This means that the converter may have to be derated if it is to operate as a negative sequence controller.

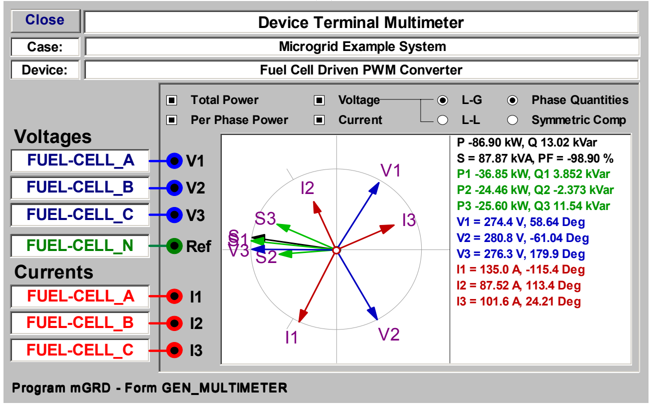


Case: Microgrid Example System
Device: Fuel Cell Driven PWM Converter

☑ Total Power ☑ Voltage ● L-G ○ Phase Quantities
☑ Per Phase Power ☑ Current ○ L-L ● Symmetric Comp

P -86.90 kW, Q 13.02 kVar
S = 87.87 kVA, PF = -98.90 %
P1 -36.85 kW, Q1 3.852 kVar
P2 -24.46 kW, Q2 -2.373 kVar
P3 -25.60 kW, Q3 11.54 kVar
V1 = 277.1 V, 59.18 Deg
V2 = 34.49 nV, -120.8 Deg
V0 = 3.779 V, -77.42 Deg
I1 = 105.7 A, -112.3 Deg
I2 = 30.03 A, -126.3 Deg
I0 = 0.101 pA, 179.0 Deg

(a)



Case: Microgrid Example System
Device: Fuel Cell Driven PWM Converter

☑ Total Power ☑ Voltage ● L-G ● Phase Quantities
☑ Per Phase Power ☑ Current ○ L-L ○ Symmetric Comp

P -86.90 kW, Q 13.02 kVar
S = 87.87 kVA, PF = -98.90 %
P1 -36.85 kW, Q1 3.852 kVar
P2 -24.46 kW, Q2 -2.373 kVar
P3 -25.60 kW, Q3 11.54 kVar
V1 = 274.4 V, 58.64 Deg
V2 = 280.8 V, -61.04 Deg
V3 = 276.3 V, 179.9 Deg
I1 = 135.0 A, -115.4 Deg
I2 = 87.52 A, 113.4 Deg
I3 = 101.6 A, 24.21 Deg

(b)

Figure 7: Voltage, current and power flow at the fuel cell PWM converter output terminals, with negative sequence control. (a) Symmetric Components, (b) Phase Quantities.

Safety. A µGrid is subject to the same safety concerns as any utility system. Any faults on the utility side may generate substantial high ground potential rise at the energy source location, even if the source may operate at low voltage (208 or 480 volts). This means that distributed energy sources must be grounded with the same rules and standards as conventional systems to achieve the same level of safety. The basic safety concern relates to the voltage at the system ground. The proposed model has the capability of computing the voltage at the grounded portions of the system and therefore can be used to study safety concerns for the μGrid.

Objectionable Currents A μGrid, by virtue of its imbalance, is subject to stray voltages and currents. The μGrid can be designed with a "common neutral", i.e. a neutral that is grounded in more than two locations. In this case, the unbalance current will split between the neutral and the ground conductors and earth, thus generating stray voltages and currents. Most of the times, stray voltages and currents are harmless. However, the μGrid is very close to the end user and therefore the human exposure to objectionable current effects is quite high. Analysis of the problem requires explicit modeling of the grounding system and grounding conductors together with the phase conductors and neutrals. The proposed model explicitly represents ground conductors and neutral conductors. Therefore the proposed method is suitable for studying objectionable currents. It is important to point out that the conditions for objectionable current depend on the use of a common neutral, i.e. a neutral grounded at more than one locations. The National Electrical Code permits this practice under certain conditions. In these cases the problem should be carefully studied.

5. Summary and Conclusions

This paper has presented a summary of unique analysis and design issues specific to μGrids. A new method is proposed for the steady state solution of the mGrid. The method has the following features:

- Full three-phase analysis
- Explicit modeling of grounding and bonding of the system, i.e. modeling of 3-wire, 4-wire and 5-wire systems, and
- Methods for modeling a variety of microsources controls

The proposed method has been applied to few applications and few examples are provided in the paper.

6. References

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Biographies

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George J. Cokkinides (M ’85) was born in Athens, Greece in 1955. He obtained the B.S., M.S., and Ph.D. degree at the Georgia Institute of Technology in 1978, 1980, and 1985, respectively. From 1983 to 1985, he was a research engineer at the Georgia Tech Research Institute. Since 1985, he has been with the university of South Carolina where he is presently an Associate Professor of Electrical Engineering. His research interests include power system modeling and simulation, power electronics applications, power system harmonics, and measurement instrumentation. Dr. Cokkinides is a member of the IEEE/PES and the Sigma Xi.

Robert H Lasseter (F ’92): Professor of Electrical Engineering, University of Wisconsin, Madison. He received his PhD in Physics from the University of Pennsylvania, Philadelphia, 1971. He was a Consultant Engineer at General Electric Co. until he joined the University of Wisconsin-Madison in 1980. Research interests focus on the application of power electronics to utility systems and technical issues which arise from the restructuring of the power utility system. This work includes interfacing micro-turbines and fuel cells to the distribution grid, control of power systems through FACTS controllers, use of power electronics in distribution systems, harmonic interactions, simulation methods, power electronic circuits and converter. He is a Fellow of IEEE and expert advisor to CIGRE SC14.